অনুশীলনী-১৪

নির্দিষ্ট যোগজ (Definite Integral)

সংক্ষিপ্ত ও রচনামূলক প্রশ্নঃ

১. $\int_0^{\pi/2} \mathbf{Sin^2xdx}$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^{\pi/2} \text{Sin}^2\text{xdx}$

$$= \frac{1}{2}.\int_0^{\pi/2} 2\text{Sin}^2\text{xdx}$$

$$= \frac{1}{2}.\int_0^{\pi/2}(1-\text{Cos}2\text{x})\text{dx}$$

$$= \frac{1}{2}\left[\text{x}-\frac{\text{Sin}2\text{x}}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{4}[2\text{x}-\text{Sin}2\text{x}]_0^{\pi/2}$$

$$= \frac{1}{4}\left[\left(2.\frac{\pi}{2}-\text{Sin}2.\frac{\pi}{2}\right)-(2.0-\text{Sin}2.0)\right]$$

$$= \frac{1}{4}[(\pi-0)-(0-0)] = \frac{\pi}{4}$$ **(Ans)**

২. $\int_0^{\pi/2} \mathbf{Cos^2xdx}$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^{\pi/2} \mathrm{Cos}^2 x dx = \frac{1}{2}.\int_0^{\pi/2} 2\mathrm{Cos}^2 x dx = \frac{1}{2}.\int_0^{\pi/2}(1+\mathrm{Cos}2x)dx = \frac{1}{2}\left[x+\frac{\mathrm{Sin}2x}{2}\right]_0^{\pi/2}$

$$= \frac{1}{4}[2x+\mathrm{Sin}2x]_0^{\pi/2} = \frac{1}{4}\left[\left(2.\frac{\pi}{2}+\mathrm{Sin}2.\frac{\pi}{2}\right)-(2.0+\mathrm{Sin}2.0)\right]$$

$$= \frac{1}{4}[(\pi+0)-(0+0)] = \frac{\pi}{4}\ \textbf{(Ans)}$$

৩. $\int_0^{\pi/2} \mathbf{Sin^3xdx}$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^{\pi/2} \mathrm{Sin}^3 x dx = \frac{1}{4}.\int_0^{\pi/2} 4\mathrm{Sin}^3 x dx = \frac{1}{4}.\int_0^{\pi/2}(3\mathrm{Sin}x-\mathrm{Sin}3x)dx$

$$= \frac{1}{4}\left[-3\mathrm{Cos}x+\frac{\mathrm{Cos}3x}{3}\right]_0^{\pi/2} = \frac{1}{12}[\mathrm{Cos}3x-9\mathrm{Cos}x]_0^{\pi/2}$$

$$= \frac{1}{12}\left[\left(\mathrm{Cos}3.\frac{\pi}{2}-9\mathrm{Cos}\frac{\pi}{2}\right)-(\mathrm{Cos}0-9\mathrm{Cos}0)\right]$$

$$= \frac{1}{12}[(0-0)-(1-9)] = \frac{2}{3}\ \textbf{(Ans)}$$

৪. $\int_0^{\pi/2} \mathbf{Cos^3xdx}$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^{\pi/2} \mathrm{Cos}^3\mathrm{x}dx = \frac{1}{4}.\int_0^{\pi/2} 4\mathrm{Cos}^3\mathrm{x}dx = \frac{1}{4}.\int_0^{\pi/2}(\mathrm{Cos3x} + 3\mathrm{Cosx})dx$

$$= \frac{1}{4}\left[\frac{\mathrm{Sin3x}}{3} + 3\mathrm{Sinx}\right]_0^{\pi/2} = \frac{1}{12}[\mathrm{Sin3x} + 9\mathrm{Sinx}]_0^{\pi/2}$$

$$= \frac{1}{12}\left[\left(\mathrm{Sin3}.\frac{\pi}{2} + 9\mathrm{Sin}\frac{\pi}{2}\right) - (\mathrm{Sin0} + 9\mathrm{Sin0})\right]$$

$$= \frac{1}{12}[(-1 + 9) - (0 + 0)] = \frac{2}{3}$$ **(Ans)**

৫. $\int_0^{\pi/6} \mathbf{Sin3xCos3xdx}$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^{\pi/6} \mathrm{Sin3xCos3x}dx = \frac{1}{2}\int_0^{\pi/6} 2\mathrm{Sin3xCos3x}dx = \frac{1}{2}\int_0^{\pi/6} \mathrm{Sin6x}dx$

$$= \frac{1}{2}\left[\frac{-\mathrm{Cos6x}}{6}\right]_0^{\pi/2} = \frac{-1}{12}(\mathrm{Cos3\pi} - \mathrm{Cos0})$$

$$= \frac{-1}{12}(-1 - 1) = \frac{1}{6}$$ **(Ans)**

৬. $\int_0^1 \mathbf{x^3\sqrt{1+3x^4}dx}$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^1 x^3\sqrt{1+3x^4}dx$

$= \int_1^4 \sqrt{z}.\frac{dz}{12} = \frac{1}{12}.\left[\frac{x^{3/2}}{3/2}\right]_1^4$

$= \frac{1}{12}.\frac{2}{3}\left[4^{3/2}-1\right] = \frac{7}{18}$ **(Ans)**

ধরি, $z = 1 + 3x^4$

বা, $\frac{dz}{dx} = 0 + 12x^3 \quad \therefore \quad \frac{dz}{12} = x^3dx$

যখন $x = 0$ তখন, $z = 1 + 3.0^4 = 1 + 0 = 1$

আবার, যখন $x = 1$ তখন, $z = 1 + 3.1^4 = 1 + 3 = 4$

৭. $\int_0^1 \mathbf{xe^{x^2}.dx}$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^1 xe^{x^2}.dx = \int_0^1 e^z.\frac{dz}{2}$

$= \frac{1}{2}\int_0^1 e^z.dz = \frac{1}{2}[e^z]_0^1$

$= \frac{1}{2}(e^1 - e^0) = \frac{1}{2}(e-1)$ **(Ans)**

ধরি, $z = x^2$

বা, $\frac{dz}{dx} = 2x$

$\therefore \frac{dz}{2} = xdx$

যখন $x = 0$ তখন, $z = 0$

আবার, যখন $x = 1$ তখন, $z = 1$

৮. $\int_0^1 \frac{(\tan^{-1}x)^2}{1+x^2}.dx$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^1 \frac{(\tan^{-1}x)^2}{1+x^2}.dx$

$= \int_0^{\pi/4} z^2 dz = \left[\frac{z^3}{3}\right]_0^{\pi/4}$

$= \frac{1}{3}\left(\frac{\pi^3}{64} - 0\right) = \frac{\pi^3}{192}$ **(Ans)**

ধরি, $z = \tan^{-1}x$

বা, $\frac{dz}{dx} = \frac{1}{1+x^2}$

$\therefore\ dz = \frac{dx}{1+x^2}$

যখন $x = 0$ তখন, $z = \tan^{-1}0 = 0$

আবার, যখন $x = 1$ তখন, $z = \tan^{-1}1 = \frac{\pi}{4}$

৯. $\int_0^1 \frac{\text{Sin}^{-1}x}{\sqrt{1-x^2}}.dx$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^1 \frac{\text{Sin}^{-1}x}{\sqrt{1-x^2}}.dx$

$= \int_0^{\pi/2} z dz = \left[\frac{z^2}{2}\right]_0^{\pi/2}$

$= \frac{1}{2}\left(\frac{\pi^2}{4} - 0\right) = \frac{\pi^2}{8}$ **(Ans)**

ধরি, $z = \text{Sin}^{-1}x$

বা, $\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\therefore\ dz = \frac{dx}{\sqrt{1-x^2}}$

যখন $x = 0$ তখন, $z = \text{Sin}^{-1}0 = 0$

আবার, যখন $x = 1$ তখন, $z = \text{Sin}^{-1}1 = \frac{\pi}{2}$

১০. $\int_0^{\pi/2} \frac{\text{Cos}^3x}{\sqrt{\text{Sinx}}}.\,dx$ এর মান নির্ণয় কর।

সমাধানঃ $\int_0^{\pi/2} \frac{\text{Cos}^3x}{\sqrt{\text{Sinx}}}.\,dx = \int_0^{\pi/2} \frac{\text{Cos}^2x}{\sqrt{\text{Sinx}}}.\,\text{Cosxdx} = \int_0^{\pi/2} \frac{(1-\text{Sinx}^2x)}{\sqrt{\text{Sinx}}}.\,\text{Cosxdx}$

এখন, $\int_0^{\pi/2} \frac{(1-\text{Sinx}^2x)}{\sqrt{\text{Sinx}}}.\,\text{Cosxdx}$

$= \int_0^1 \left(\frac{1-z^2}{\sqrt{z}}\right) dz = \int_0^1 \left(\frac{1}{\sqrt{z}} - z^{3/2}\right) dz$

$= \left[2\sqrt{z} - \frac{z^{5/2}}{5/2}\right]_0^1 = \left[2\sqrt{z} - \frac{2}{5}.\,z^{5/2}\right]_0^1$

$= \left(2 - \frac{2}{5}\right) - 0 = \frac{8}{5}$ **(Ans)**

ধরি, $z = \text{Sinx}$

বা, $\frac{dz}{dx} = \text{Cosx}$

$\therefore\ dz = \text{Cosxdx}$

যখন $x = 0$ তখন, $z = \text{Sin}0 = 0$

আবার, যখন $x = \frac{\pi}{2}$ তখন, $z = \text{Sin}\frac{\pi}{2} = 1$

Thanks All